

Rare Z-decay into light pseudoscalar bosons in the simplest little Higgs model

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Abstract

The simplest little Higgs model predicts a light pseudoscalar boson η and opens up some new decay modes for Z -boson, such as $Z \rightarrow \bar{f}f\eta$, $Z \rightarrow \eta\eta\eta$, $Z \rightarrow \eta\gamma$ and $Z \rightarrow \eta gg$. We examine these decay modes in the parameter space allowed by current experiments, and find that the branching ratios can reach 10^{-7} for $Z \rightarrow \bar{b}b\eta$, 10^{-8} for $Z \rightarrow \bar{\tau}\tau\eta$, and 10^{-8} for $Z \rightarrow \eta\gamma$, which should be accessible at the GigaZ option of the ILC. However, the branching ratios can reach 10^{-12} for $Z \rightarrow \eta\eta\eta$, and 10^{-11} for $Z \rightarrow \eta gg$, which are hardly accessible at the GigaZ option.

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I. INTRODUCTION

Little Higgs theory [1] has been proposed as an interesting solution to the hierarchy problem. So far various realizations of the little Higgs symmetry structure have been proposed [2–5], which can be categorized generally into two classes [6]. One class use the product group, represented by the littlest Higgs model [3], in which the SM $SU(2)_L$ gauge group is from the diagonal breaking of two (or more) gauge groups. The other class use the simple group, represented by the simplest little Higgs model (SLHM) [4], in which a single larger gauge group is broken down to the SM $SU(2)_L$.

Since these little Higgs models mainly alter the properties of the Higgs boson, hints of these models may be unraveled from various Higgs boson processes. The phenomenology of Higgs boson in these little Higgs models has been widely studied [7–10]. In addition to the SM-like Higgs boson h , the SLHM predicts a pseudoscalar boson η , whose mass can be as low as $\mathcal{O}(10)$ GeV. The constraint from the non-observation in the decay $\Upsilon \rightarrow \gamma\eta$ excludes η with mass below 5–7 GeV [11]. Thus, the Z -decays into η can be open for a light η , such as $Z \rightarrow \bar{f}f\eta$ ($f = b, \tau$), $Z \rightarrow \eta\eta\eta$, $Z \rightarrow \eta\gamma$ and $Z \rightarrow \eta gg$.

The next generation Z factory can be realized in the GigaZ option of the International Linear Collider (ILC)[12]. The ILC is a proposed electron-positron collider with tunable energy ranging from 400GeV to 500GeV and polarized beams in its first phase, and the GigaZ option corresponds to its operation on top of the resonance of Z -boson by adding a bypass to its main beam line. About 2×10^9 Z events can be generated in an operational year of 10^7 s of GigaZ, which implies that the expected sensitivity to the branching ratio of Z -decay can be improved from 10^{-5} at the LEP to 10^{-8} at the GigaZ [12]. Therefore, it will offer an important opportunity to probe the new physics via the exotic or rare decays of Z -boson. The Z -boson flavor-changing neutral-current (FCNC) decays have been studied in many new physics models. For the lepton (quark) flavor violation decays, the branching ratios can be enhanced sizably compared with the SM predictions [13, 14]. In addition to the FCNC processes, the branching ratios for the processes of the Z -decay into light Higgs boson(s) in some new physics models can be accessible at the GigaZ option of the ILC [15–17]. In this paper, we will focus on the processes of the Z -decays into η in the SLHM, namely $Z \rightarrow \bar{f}f\eta$ ($f = b, \tau$), $Z \rightarrow \eta\eta\eta$, $Z \rightarrow \eta\gamma$ and $Z \rightarrow \eta gg$.

This work is organized as follows. In Sec. II we recapitulate the SLHM. In Sec. III we

study the five decay modes, respectively. Finally, we give our conclusion in Sec. IV.

II. SIMPLEST LITTLE HIGGS MODEL

The SLHM is based on $[SU(3) \times U(1)_X]^2$ global symmetry. The gauge symmetry $SU(3) \times U(1)_X$ is broken down to the SM electroweak gauge group by two copies of scalar fields Φ_1 and Φ_2 , which are triplets under the $SU(3)$ with aligned VEVs f_1 and f_2 . The uneaten five pseudo-Goldstone bosons can be parameterized as

$$\Phi_1 = e^{i t_\beta \Theta} \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix}, \quad \Phi_2 = e^{-\frac{i}{t_\beta} \Theta} \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix}, \quad (1)$$

where

$$\Theta = \frac{1}{f} \left[\begin{pmatrix} 0 & 0 & H \\ 0 & 0 & H^\dagger \\ H^\dagger & 0 \end{pmatrix} + \frac{\eta}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right], \quad (2)$$

$f = \sqrt{f_1^2 + f_2^2}$ and $t_\beta \equiv \tan \beta = f_2/f_1$. Under the $SU(2)_L$ SM gauge group, η is a real scalar, while H transforms as a doublet and can be identified as the SM Higgs doublet. The kinetic term in the non-linear sigma model is

$$\mathcal{L}_\Phi = \sum_{j=1,2} \left| \left(\partial_\mu + ig A_\mu^a T^a - i \frac{g_x}{3} B_\mu^x \right) \Phi_j \right|^2, \quad (3)$$

where $g_x = g \tan \theta_W / \sqrt{1 - \tan^2 \theta_W / 3}$ with θ_W being the electroweak mixing angle. As Φ_1 and Φ_2 develop their VEVs, the new heavy gauge bosons Z' , Y^0 , and X^\pm get their masses proportional to f . A novel coupling of the SM-like Higgs boson h can be derived from the Eq. (3),

$$\mathcal{L}_{Zh\eta} = \frac{m_Z}{\sqrt{2}f} (t_\beta - \frac{1}{t_\beta}) (\eta \partial^\mu h - h \partial^\mu \eta). \quad (4)$$

The gauged $SU(3)$ symmetry promotes the SM fermion doublets into $SU(3)$ triplets. There are two possible gauge charge assignments for the fermions: the 'universal' embedding and the 'anomaly-free' embedding. The first choice is not favored by the electroweak precision data [4], so we focus on the second way of embedding. The quark Yukawa interactions for the third generation and the first two generations can be written respectively

as

$$\mathcal{L}_3 = i\lambda_1^t t_1^c \Phi_1^\dagger Q_3 + i\lambda_2^t t_2^c \Phi_2^\dagger Q_3 + i\frac{\lambda_d^m}{\Lambda} d_m^c \epsilon_{ijk} \Phi_1^i \Phi_2^j Q_3^k + h.c., \quad (5)$$

$$\mathcal{L}_{1,2} = i\lambda_1^{d_n} d_{1n}^c Q_n^T \Phi_1 + i\lambda_2^{d_n} d_{2n}^c Q_n^T \Phi_2 + i\frac{\lambda_u^{mn}}{\Lambda} u_m^c \epsilon_{ijk} \Phi_1^{*i} \Phi_2^{*j} Q_n^k + h.c., \quad (6)$$

where $n = 1, 2$ are the first two generation indices; $i, j, k = 1, 2, 3$; $Q_3 = \{t_L, b_L, iT_L\}$ and $Q_n = \{d_{nL}, -u_{nL}, iD_{nL}\}$; d_m^c runs over $(d^c, s^c, b^c, D^c, S^c)$; d_{1n}^c and d_{2n}^c are linear combinations of d^c and D^c for $n = 1$ and of s^c and S^c for $n = 2$; u_m^c runs over (u^c, c^c, t^c, T^c) . For simplicity, we assume the quark flavor mixing are small and neglect the mixing effects. Eqs. (5) and (6) contain the Higgs boson interactions and the mass terms for the three generations of quarks:

$$\mathcal{L}_t \simeq -f\lambda_2^t [x_\lambda^t c_\beta t_1^c (-s_1 t_L + c_1 T_L) G_1(\eta) + s_\beta t_2^c (s_2 t_L + c_2 T_L) G_2(\eta)] + h.c., \quad (7)$$

$$\mathcal{L}_{d_n} \simeq -f\lambda_2^{d_n} [x_\lambda^{d_n} c_\beta d_1^c (s_1 d_{nL} + c_1 D_{nL}) G_1^*(\eta) + s_\beta d_2^c (-s_2 d_{nL} + c_2 D_{nL}) G_2^*(\eta)] + h.c., \quad (8)$$

$$\mathcal{L}_b \simeq -\frac{\lambda_b}{\Lambda} f^2 s_\beta c_\beta s_3 b_L G_3(\eta) + h.c., \quad (9)$$

$$\mathcal{L}_q \simeq -\frac{\lambda_q}{\Lambda} f^2 s_\beta c_\beta s_3 q_L G_3^*(\eta) + h.c. \quad (q = u, c), \quad (10)$$

where

$$\begin{aligned} x_\lambda^t &\equiv \frac{\lambda_1^t}{\lambda_2^t}, \quad x_\lambda^{d_n} \equiv \frac{\lambda_1^{d_n}}{\lambda_2^{d_n}}, \quad s_\beta \equiv \frac{f_2}{\sqrt{f_1^2 + f_2^2}}, \quad c_\beta \equiv \frac{f_1}{\sqrt{f_1^2 + f_2^2}}, \\ s_1 &\equiv \sin \frac{t_\beta(h+v)}{\sqrt{2}f}, \quad s_2 \equiv \sin \frac{(h+v)}{\sqrt{2}t_\beta f}, \quad s_3 \equiv \sin \frac{(h+v)(t_\beta^2 + 1)}{\sqrt{2}t_\beta f}, \\ G_1(\eta) &\equiv 1 - i\frac{t_\beta}{\sqrt{2}f}\eta - \frac{t_\beta^2}{4f^2}\eta^2, \quad G_2(\eta) \equiv 1 + i\frac{1}{\sqrt{2}t_\beta f}\eta - \frac{1}{4t_\beta^2 f^2}\eta^2, \\ G_3(\eta) &\equiv 1 + i\frac{1}{\sqrt{2}f}(t_\beta - \frac{1}{t_\beta})\eta - \frac{1}{4f^2}(t_\beta - \frac{1}{t_\beta})^2\eta^2, \end{aligned} \quad (11)$$

with h and v being the SM-like Higgs boson field and its VEV, respectively. The mass eigenstates are obtained by mixing the corresponding interaction eigenstates, e.g., the mass eigenstates (t_{mL}, T_{mL}) and (t_m^c, T_m^c) are respectively the mixtures of (t_L, T_L) and (t^c, T^c) . The diagonalization of the mass matrix in Eqs. (7) and (8) is performed numerically in our analysis, and the top quark (T, D, d, S, s) couplings of h and η bosons can also be obtained without resort to any expansion of v/f . From Eqs. (9) and (10), we can get directly the couplings $-i\frac{m_f}{v}x_f h\bar{f}f$ and $\frac{m_f}{v}y_f \eta\bar{f}\gamma_5 f$ ($f = b, u, c$) with

$$x_f = \frac{v}{\sqrt{2}f}(t_\beta + \frac{1}{t_\beta}) \cot \frac{v}{\sqrt{2}f}(t_\beta + \frac{1}{t_\beta}), \quad y_u = y_c = -y_b = \frac{v}{\sqrt{2}f}(t_\beta - \frac{1}{t_\beta}). \quad (12)$$

The charged lepton couplings with h and η are the same as those of b quark, but replacing m_b with the corresponding lepton mass. Hereafter we denote the mass eigenstates without the subscript 'm' for simplicity.

The Yukawa and gauge interactions break the global symmetry and then provide a potential for the Higgs boson. However, the Coleman-Weinberg potential alone is not sufficient since the generated h mass is too heavy and the new pseudoscalar η is massless. Therefore, one can introduce a tree-level μ term which can partially cancel the h mass [4, 9]:

$$-\mu^2(\Phi_1^\dagger\Phi_2 + h.c.) = -2\mu^2f^2s_\beta c_\beta \cos\left(\frac{\eta}{\sqrt{2}s_\beta c_\beta f}\right) \cos\left(\frac{\sqrt{H^\dagger H}}{fc_\beta s_\beta}\right). \quad (13)$$

The Higgs potential becomes

$$V = -m^2H^\dagger H + \lambda(H^\dagger H)^2 - \frac{1}{2}m_\eta^2\eta^2 + \lambda'H^\dagger H\eta^2 + \dots, \quad (14)$$

where

$$m^2 = m_0^2 - \frac{\mu^2}{s_\beta c_\beta}, \quad \lambda = \lambda_0 - \frac{\mu^2}{12s_\beta^3 c_\beta^3 f^2}, \quad \lambda' = -\frac{\mu^2}{4f^2 s_\beta^3 c_\beta^3} \quad (15)$$

with m_0 and λ_0 being respectively the one-loop contributions to the H mass and the quartic couplings from the contributions of fermion loops and gauge boson loops [4].

After EWSB, the coupling $h\eta\eta$ can be obtained from the term $\lambda'H^\dagger H\eta^2$ in Eq. (14). The Higgs VEV and the masses of h and η are given by

$$v^2 = \frac{m^2}{\lambda}, \quad m_h^2 = 2m^2, \quad m_\eta^2 = \frac{\mu^2}{s_\beta c_\beta} \cos\left(\frac{v}{\sqrt{2}f s_\beta c_\beta}\right). \quad (16)$$

The Coleman-Weinberg potential involves the following parameters:

$$f, x_\lambda^t, t_\beta, \mu, m_\eta, m_h, v. \quad (17)$$

Due to the modification of the observed W gauge boson mass, v is defined as [9]

$$v \simeq v_0 \left[1 + \frac{v_0^2}{12f^2} \frac{t_\beta^4 - t_\beta^2 + 1}{t_\beta^2} - \frac{v_0^4}{180f^4} \frac{t_\beta^8 - t_\beta^6 + t_\beta^4 - t_\beta^2 + 1}{t_\beta^4} \right], \quad (18)$$

where $v_0 = 246$ GeV is the SM Higgs VEV. Assuming that there are no large direct contributions to the potential from physics at the cutoff, we can determine other parameters in Eq. (17) from f , t_β and m_η with the definition of v in Eq. (18).

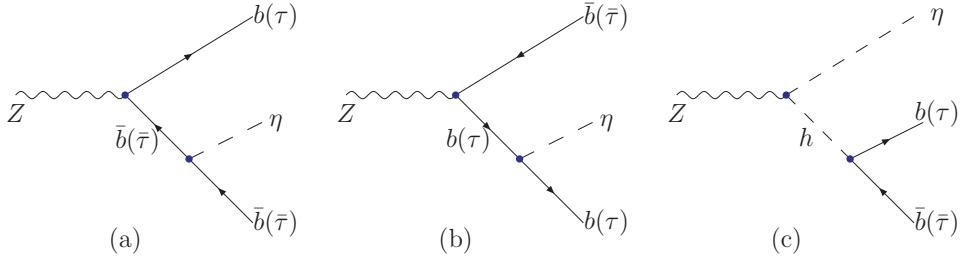


FIG. 1. Feynman diagrams for $Z \rightarrow \bar{f}f\eta$ ($f = b, \tau$) in the SLHM.

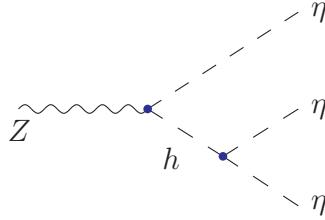


FIG. 2. Feynman diagrams for $Z \rightarrow \eta\eta\eta$ in the SLHM.

III. THE RARE Z-DECAY INTO η

In the SLHM, the rare Z -decays $Z \rightarrow \bar{f}f\eta$ ($f = b, \tau$), $Z \rightarrow \eta\eta\eta$, $Z \rightarrow \eta\gamma$ and $Z \rightarrow \eta gg$ can be depicted by the Feynman diagrams shown in Fig. 1, Fig. 2, Fig. 3 and Fig. 4, respectively. For the decay $Z \rightarrow \eta\eta\eta$, ref. [15] shows the contributions of the scalar-loop diagrams can also be important for the large $h\eta\eta$ coupling. Here we do not consider the contributions of the loop diagram since the $h\eta\eta$ coupling does not have the large factor of enhancement in the SLHM (see λ' in Eq.(15)).

The calculations of the loop diagrams in Fig. 3 and Fig. 4 are straightforward. Each loop diagram is composed of some scalar loop functions [18] which are calculated by using LoopTools [19]. In appendix A, we list the amplitudes for $Z \rightarrow \bar{f}f\eta$, $Z \rightarrow \eta\eta\eta$ and $Z \rightarrow \eta\gamma$, respectively. The expressions for the amplitude of $Z \rightarrow \eta gg$ are very lengthy, which are not presented here.

In our calculations, we take $m_t = 173.3$ GeV [20] and the other SM input parameters as ref. [21]. The free SLHM parameters are f , t_β , m_η , x_λ^d and x_λ^s . As shown above, the parameters x_λ^t , μ , m_h can be determined by f , t_β , m_η and v . To satisfy the bound of LEP2, we require that m_h is larger than 114.4 GeV [22]. Certainly, due to the suppression of hZZ coupling [9], the LEP2 bound on m_h should be loosened to some extent. The recent studies about Z leptonic decay and $e^+e^- \rightarrow \tau^+\tau^-\gamma$ process at the Z -pole show that the scale f

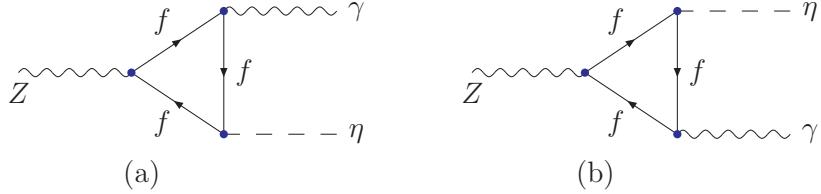


FIG. 3. Feynman diagrams for $Z \rightarrow \eta\gamma$ in the SLHM. f denotes the charged fermions in SM, the new quarks T , D , and S .

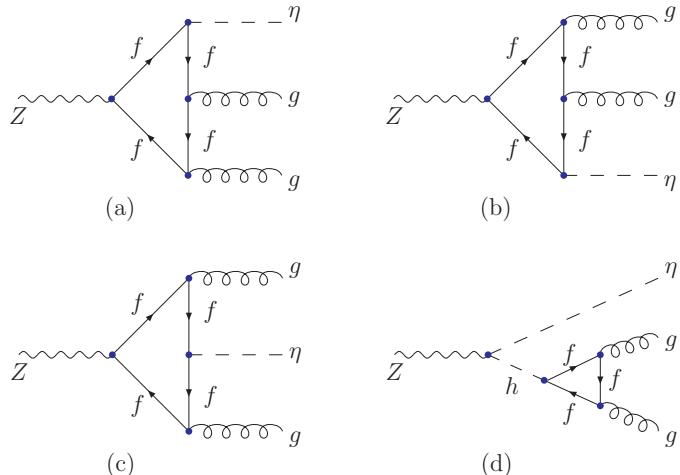


FIG. 4. Feynman diagrams for $Z \rightarrow \eta gg$ in the SLHM. f denotes the SM quarks, the new quarks T , D , and S . The diagrams by exchanging the two gluons are not shown here.

should be respectively larger than 5.6 TeV and 5.4 TeV [23]. Here, we assume the new flavor mixing matrices in lepton and quark sectors are diagonal [6, 24, 25], so that f and t_β are free from the experimental constraints of the lepton and quark flavor violating processes. The large values of f can suppress the SLHM predictions sizably. However, the factor t_β in the couplings of h and η can be taken as a large value to cancel the suppression of f partially. For the perturbation to be valid, t_β cannot be too large for a fixed f . If we require $\mathcal{O}(v_0^4/f^4)/\mathcal{O}(v_0^2/f^2) < 0.1$ in the expansion of v , t_β should be below 28 for $f = 5.6$ TeV. In our calculations, we take $f = 5.6$ TeV and $t_\beta = 28, 26, 24$, respectively.

The small mass of the $d(s)$ quark requires one of the couplings λ_1^d and λ_2^d (λ_1^s and λ_2^s) to be very small, so there is almost no mixing between the SM down-type quarks and their heavy partners. We assume $\lambda_1^d(\lambda_1^s)$ is small, and take $x_\lambda^d = 1.1 \times 10^{-4}$ ($x_\lambda^s = 2.1 \times 10^{-3}$), which can make the masses of D and S be in the range of 1 TeV and 2 TeV for other parameters taken in our calculations. In fact, our results show that different choices of x_λ^d and x_λ^s can not have sizable effects on the result.

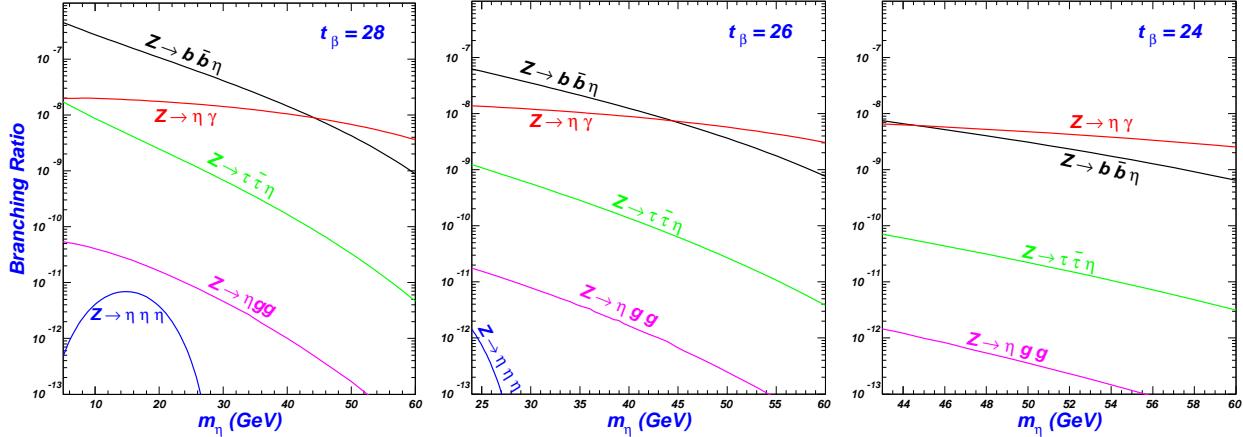


FIG. 5. For $f = 5.6$ TeV, the branching ratios of $Z \rightarrow \bar{f}f\eta$ ($f = b, \tau$), $Z \rightarrow \eta\eta\eta$, $Z \rightarrow \eta\gamma$ and $Z \rightarrow \eta gg$ versus the η boson mass. The incomplete lines for $t_\beta = 26$ and 24 show the lower bounds of the η mass, respectively.

In Fig. 5, we plot the decay branching ratios of $Z \rightarrow \bar{f}f\eta$ ($f = b, \tau$), $Z \rightarrow \eta\eta\eta$, $Z \rightarrow \eta\gamma$ and $Z \rightarrow \eta gg$ versus the η mass for $t_\beta = 28, 26, 24$, respectively. Because of the constraint of $m_h > 114.4$ GeV, the lower bound of m_η is enhanced for the small t_β . Fig. 5 shows that the ratios can reach 7×10^{-12} for $Z \rightarrow \eta\eta\eta$ with $m_\eta = 15$ GeV and $t_\beta = 28$, and 5×10^{-11} for $Z \rightarrow \eta gg$ with $m_\eta = 5$ GeV and $t_\beta = 28$, which are too small to be detectable at the Gigaz option of the ILC. The reason should be partly from the suppression of three-body phase space. Besides, for the decay $Z \rightarrow \eta gg$, we find there is a strong cancellation between the contributions of different diagrams shown in Fig. 4, which can reduce the branching ratio sizably. As the increasing of m_η , the phase space of $Z \rightarrow \eta\eta\eta$ is suppressed, and the $h\eta\eta$ coupling which is proportional to m_η^2 is enhanced, so the ratio reaches its peak for $m_\eta = 15$ GeV and $t_\beta = 28$. In addition to the couplings $Z\eta h$ and $h\eta\eta$ which are suppressed by the scale f , the factor $\frac{1}{\hat{s} - m_h^2}$ contributed by the intermediate state h shown in Fig. 2 (see Eq. (A4)) can suppress the branching ratio of $Z \rightarrow \eta\eta\eta$ sizably.

Fig. 5 shows that the ratios can reach 10^{-7} for $Z \rightarrow \eta\bar{b}b$, 10^{-8} for $Z \rightarrow \eta\bar{\tau}\tau$, and 10^{-8} for $Z \rightarrow \eta\gamma$ for a light η boson with $t_\beta = 28$, which should be accessible at the Gigaz option. The three ratios decrease with increasing of m_η , especially for the decays $Z \rightarrow \eta\bar{b}b$ and $Z \rightarrow \eta\bar{\tau}\tau$. For $t_\beta = 26$, the lower bound of m_η is enhanced to 24 GeV to satisfy the constraint of $m_h > 114.4$ GeV, and the ratios of $Z \rightarrow \eta\bar{\tau}\tau$ is below 10^{-8} , which is hardly accessible at the Gigaz option. For $t_\beta = 24$, the lower bound of m_η is enhanced to 43 GeV, and the ratios of the three decays are too small to detectable at the Gigaz option. The

branching ratio of $Z \rightarrow \eta \bar{b}b$ always dominates over $Z \rightarrow \eta \bar{\tau}\tau$ since the coupling $\eta \bar{b}b$ is larger than $\eta \bar{\tau}\tau$, and the former can be enhanced by the color factor. Although the decay $Z \rightarrow \eta \gamma$ occurs at one-loop level, the branching ratio can be comparable with those of $Z \rightarrow \eta \bar{f}f$ because of the large couplings $\eta \bar{t}t$ and $\eta \bar{T}T$, and suppression of the three-body phase space for $Z \rightarrow \eta \bar{f}f$.

IV. CONCLUSION

In the framework of the simplest little Higgs model, we studied the rare Z-decays $Z \rightarrow \bar{f}f\eta$, $Z \rightarrow \eta\eta\eta$, $Z \rightarrow \eta\gamma$ and $Z \rightarrow \eta gg$. In the parameter space allowed by current experiments, the branching ratios can reach 10^{-7} for $Z \rightarrow \bar{b}b\eta$, 10^{-8} for $Z \rightarrow \bar{\tau}\tau\eta$, and 10^{-8} for $Z \rightarrow \eta\gamma$, which should be accessible at the GigaZ option of the ILC. However, the branching ratios can reach 10^{-12} for $Z \rightarrow \eta\eta\eta$, and 10^{-11} for $Z \rightarrow \eta gg$, which are too small to be detectable at the Gigaz.

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Appendix A: Amplitudes of $Z \rightarrow \bar{f}f\eta$, $Z \rightarrow \eta\eta\eta$ and $Z \rightarrow \eta\gamma$

Here we give the amplitude of the process $Z(p_1, \epsilon_1) \rightarrow \bar{f}(p_4)f(p_3)\eta(p_2)$ ($f = b, \tau$). The expressions are given by

$$M = M_a + M_b + M_c, \quad (\text{A1})$$

where

$$\begin{aligned} M_a &= -\frac{1}{(p_1 - p_4)^2 - m_f^2} \bar{u}(p_3)(g_L^\eta P_L + g_R^\eta P_R)(\not{p}_1 - \not{p}_4 + m_f)\gamma^\mu(g_L^Z P_L + g_R^Z P_R)v(p_4)\epsilon_1(p_1), \\ M_b &= -\frac{1}{(p_1 - p_3)^2 - m_f^2} \bar{u}(p_3)\gamma^\mu(g_L^Z P_L + g_R^Z P_R)(\not{p}_3 - \not{p}_1 + m_f)(g_L^\eta P_L + g_R^\eta P_R)v(p_4)\epsilon_1(p_1), \\ M_c &= \frac{g_{hf}\bar{f}g_{Zh\eta}}{(p_3 + p_4)^2 - m_h^2} \bar{u}(p_3)v(p_4)(p_1 - 2p_3 - 2p_4)^\mu\epsilon_\mu(p_1). \end{aligned} \quad (\text{A2})$$

The amplitude of $Z(p_1, \epsilon_1) \rightarrow \eta(p_2)\eta(p_3)\eta(p_4)$ is as follows:

$$M = M_1(p_1, p_2, p_3, p_4) + M_1(p_1, p_3, p_2, p_4) + M_1(p_1, p_4, p_3, p_2), \quad (\text{A3})$$

where

$$M_1(p_1, p_2, p_3, p_4) = \frac{g_{h\eta\eta}g_{Zh\eta}}{\hat{s} - m_h^2}(p_3 + p_4 - p_2)^\mu \epsilon_\mu(p_1) \text{ with } \hat{s} = (p_3 + p_4)^2. \quad (\text{A4})$$

The expressions for the amplitude of $Z(p_1, \epsilon_1) \rightarrow \eta(p_3)\gamma(p_2, \epsilon_2)$ can be given by

$$\begin{aligned} M = & \frac{N_{cf}Q_f}{4\pi^2}(A + B)[\epsilon_1 \cdot \epsilon_2^*(-2m_f C_\nu p_2^\nu + \frac{m_f}{2} - m_f C_{\mu\nu} g^{\mu\nu} + m_f p_2 \cdot p_3 C_0 + m_f^3 C_0) \\ & + p_2 \cdot \epsilon_2^*(2m_f C_\mu \epsilon_1^\mu - m_f p_3 \cdot \epsilon_1 C_0) + p_2 \cdot \epsilon_1(2m_f C_\nu \epsilon_2^{\nu*} - m_f p_3 \cdot \epsilon_2^* C_0) \\ & - 2m_f p_3 \cdot \epsilon_1 C_\nu \epsilon_2^{\nu*} + 4m_f C_{\mu\nu} \epsilon_1^\mu \epsilon_2^{\nu*}] + i \frac{N_{cf}Q_f}{4\pi^2}(C - D)m_f \epsilon_{\nu\alpha\mu\beta} \epsilon_1^\mu \epsilon_2^{\nu*} p_2^\alpha p_3^\beta C_0, \end{aligned} \quad (\text{A5})$$

where N_{cf} and Q_f denote the color factor and the electric charge for the fermion f , respectively; $A = \frac{1}{2}(g_L^Z g_R^\eta + g_R^Z g_L^\eta)$, $B = \frac{1}{2}(g_L^Z g_L^\eta + g_R^Z g_R^\eta)$, $C = \frac{1}{2}(g_L^Z g_R^\eta - g_R^Z g_L^\eta)$, and $D = \frac{1}{2}(g_L^Z g_L^\eta - g_R^Z g_R^\eta)$. The parameters g_L^Z , g_R^Z , g_L^η and g_R^η are respectively from the couplings $i\bar{f}\gamma^\mu(g_L^Z P_L + g_R^Z P_R)fZ$ and $\bar{f}(g_L^\eta P_L + g_R^\eta P_R)f\eta$ with $P_{L,R} = \frac{1 \mp \gamma^5}{2}$. C_0 , C_μ and $C_{\mu\nu}$ are respectively the 3-point loop functions, and their dependence is given by $C_{0,\mu,\mu\nu} \equiv C_{0,\mu,\mu\nu}(p_2, -p_1, m_f, m_f, m_f)$.

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